

# A Wave Optimized Adaptive, High-Order, Multi-Domain Method for Parallel Architectures

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# A Wavelet Optimized Adaptive, High-Order, Multi-Domain Method for Parallel Architectures

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Constructing numerical schemes which are both adaptive and suitable for parallel architectures is very challenging. The challenge lies in the need to maintain a balanced load across the processing elements using a method that is both efficient and scalable. Here we propose a method which is adaptive, load balanced, absolutely efficient and scalable offering significant speedup over lower order adaptive schemes.

The ability of wavelets to accurately and efficiently represent functions with localized features has spawned intensive research into applying wavelets for the solution of partial differential equations with the promise of significantly reducing the necessary computational effort and memory requirements. Traditionally, this effort has been centered around using wavelets as an orthogonal and complete basis, spanning a space in which to seek approximate solutions satisfying the equation in a Galerkin sense. Besides from the well known difficulties associated with such an approach for non-linear problems, one is also faced with the problem of dealing with non-trivial boundary conditions in an accurate and stable manner.

Such restrictions on the applicability of wavelet based methods for the solution of problems of more general interest have, in recent years, induced significant interest into grid-based collocation wavelet methods, with various different approaches being taken. The formulation and implementation of multi-dimensional pure wavelet collocation methods, however, remains a challenging task and many issues require attention.

In the present work we take a somewhat different approach to arrive at a grid based method utilizing the unique properties of wavelets. Rather than using the wavelets as a basis, we utilize the ability of wavelets to not only detect the existence of high-frequency information but also to supply information about the spatial location of such strongly inhomogeneous regions.

<b>4x32</b>	<b>8x16</b>	<b>4x32</b>	<b>4x32</b>
<b>8x16</b>	<b>8x16</b>	<b>4x32</b>	<b>2x64</b>
<b>8x16</b>	<b>8x16</b>	<b>4x32</b>	<b>2x64</b>

Figure 1: Order times grid points for each direction.

Let us, however, for a minute simply claim that wavelets provide the proper tool for the formulation of adaptive, arbitrary grid finite difference schemes and consider the difficulties associated

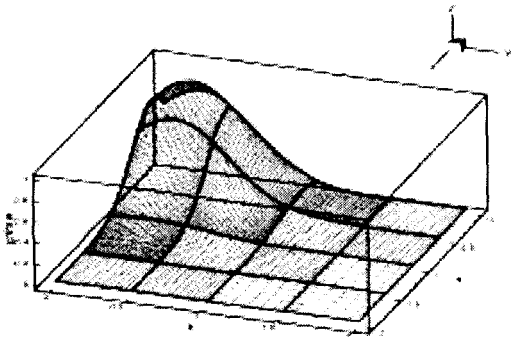


Figure 2: Propagation of Pulse for Multidomain method

with taking such an approach. Besides from obvious difficulties associated with implementing an arbitrary grid and order multi-dimensional finite difference method, finite difference schemes defined on arbitrary grids are known to introduce numerical artifacts, resulting in an amplification of numerical noise and, as a consequence, makes coarsening in smooth regions of the solution a less than trivial task, in particular when considering the use of high-order methods. Moreover, it is well known that wavelets are best suited for application on equidistant grids which, for problems beyond one dimension, suggests a tensor-product approximation. This, on the other hand, makes the application of such methods hard for problems in complex domains.

The requirement for a somewhat structured grid, while maintaining the need for geometric flexibility, points towards the introduction of a multi-domain formulation as the proper way of progressing. Indeed, as has been realized over the last decade within the community of spectral methods, multi-domain methods alleviate many of the problems associated with the use of high order methods in complex geometries, while, for many problems, providing the computationally most efficient framework in which to solve a multitude of problems of more general interest. In this work, we propose to combine the geometric flexibility and computational efficiency of a multi-domain scheme with the adaptivity, facilitated by the wavelet analysis and the associated finite difference operators, to arrive at a scheme which, as we shall see, circumvents most of the problems discussed above while, at the same time, providing a very natural data-decomposition and a mechanism for load-balancing within a parallel framework.

In Figure (1) we can see an example of the order of the scheme times the number of grid points in each directions. This value, which we are calling the computational granularity, is constant in each domain yielding a perfectly load balanced system, (ignoring hardware irregularities). For large problem sizes the scheme is absolutely efficient with efficiency remaining constant as the number of processors increases or decreases. It is linearly scalable with a lower bound as the total computation time per node approaches the time for communication (typically this will be on the order of a few hundred grid points per node). The method also offers a significant speedup over other lower order adaptive schemes. In Figure (2) we can see a Gaussian pulse propagated diagonally across the domain. One can see the changing grid point density. As the grid becomes more dense the computational order is decreased thereby keeping the work in each domain fixed.

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